pages	$1-3$	$4-6$	$7-8$	$9-10$	$11-12$	$13-15$	*total*
scores							

Exam #2, April 6, Linear Algebra, Spring, 2004, W. Stephen Wilson

Name: ____________________

TA Name and section: ____________________

NO CALCULATORS, NO PAPERS, SHOW WORK.

When we work in the linear space of polynomials of degree less than or equal to n, P_n, we consider it as a subspace of the inner product space $\mathcal{C}[0,1]$ with inner product $< f, g >= \int_0^1 f(x)g(x)dx$.

(1) (3 points) Show that the set $V \subset P_3 \subset \mathcal{C}[0,1]$ of all $f \in P_3$ with $f(0) = 0 = f(1)$ is a subspace.

(2) (3 points) Find a basis for the subspace V of problem 1.

(3) (3 points) Is $x \in P_3$ in $V^{\perp}$? (Use the V of problem 1. Show work.)

(4) (3 points) Define a map $L : P_3 \to P_3 \subset \mathcal{C}[0,1]$ by $L(f) = f'(x) + x^2 \int_0^1 f(x)dx$. Show L is a linear transformation.

(5) (3 points) Using the basis $\{1, x, x^2, x^3\}$ for P_3, find the matrix for L of problem 4 with respect to this basis.

(6) (3 points) Find a basis for the kernel of the map L of problem 4.

(7) (3 points) Find an orthonormal basis for the subspace $V \subset \mathbb{R}^3$ spanned by $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

(8) (3 points) Find a basis for the orthogonal complement of the V of problem 7.

(9) (3 points) If the determinant of (v_1, v_2, v_3) is 5, then what is the determinant of $(v_2, v_1 + 3v_3, v_1 + 2v_2)$?

(10) (3 points) State Cramer's rule.

(11) (3 points) Give the formula for the inverse of A using the adjoint. Define all the terms.

(12) (3 points) Find the area of the parallelogram with two sides given by the vectors $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

(13) (3 points) Find an orthonormal basis for the subspace $V \subset P_2 \subset \mathcal{C}[0,1]$ spanned by x and x^2.

(14) (3 points) Find the orthogonal projection of the function $f(x) = 1 \in P_2$ to the subspace V of problem 13.

(15) (3 points) If you successfully did problem 14 then what you have done is found the closest parabola going through the origin to the constant function 1 on the interval $[0, 1]$. Graph it to see if your answer is reasonable.