Exam #1, Feb. 24, Linear Algebra, Spring, 2004, W. Stephen Wilson

Name: ________________________________

(0) (3 points) TA Name and section: ________________________________

NO CALCULATORS, SHOW ALL WORK, NO SCRAP PAPER.

For the first 9 problems we let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation given by orthogonal projection to the line spanned by $(1, 2)$.

(1) (3 points) Find a basis for the image subspace of T.

(2) (3 points) Find a basis for the kernel subspace of T.

(3) (3 points) Find the 2×2 matrix for T.

(4) (3 points) Show that $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ form a basis for $\mathbb{R}^2$. Call this basis $\mathcal{B}$.

(5) (3 points) Find the change of basis matrix S such that $S([\vec{x}]_{\mathcal{B}}) = \vec{x}$.

(6) (3 points) Find the inverse of the matrix S in problem (5).

(7) (3 points) Find the matrix, B, for T (of problem (1)) with respect to our new basis, $\mathcal{B}$.

(8) (3 points) Find a basis for the kernel of T written in terms of our new coordinates coming from the basis $\mathcal{B}$.

(9) (3 points) Find a basis for the image of T written in terms of our new coordinates coming from the basis $\mathcal{B}$.

Consider the matrix $A = \begin{pmatrix} 1 & 2 & -5 \\ 2 & -1 & 0 \\ 1 & 2 & -5 \end{pmatrix}$. We use this matrix for the rest of the exam.

(10) (3 points) Find the reduced row echelon form of A (i.e. rref(A)).

(11) (3 points) What is the rank of A?

(12) (3 points) Calculate $A^2 = AA$.

(13) (3 points) Calculate a basis for the kernel of A.

(14) (3 points) Calculate a basis for the image of A.

(15) (3 points) What is the rank of A^2?

(16) (3 points) Calculate a basis for the kernel of A^2.

(17) (3 points) Calculate a basis for the image of A^2.

(18) (3 points) Kernel $A \cap$ image A is a subspace of $\mathbb{R}^3$. What is its dimension?

(19) (3 points) Find a basis for the subspace Kernel $A \cap$ image A of $\mathbb{R}^3$.