Exam #2, Nov. 11, Linear Algebra, Fall, 2003, W. Stephen Wilson

Name: ______________________________

(3 points) TA Name and section: ______________________________

NO CALCULATORS, NO PAPERS, SHOW WORK.

For the first 3 problems we will work in the linear space of 2×2 matrices. We have an inner product given by: $\langle A, B\rangle = \text{trace}(A^T B)$ where the trace is the sum of the diagonal elements.

(1) (3 points) What is the length of the vector $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$?

(2) (3 points) Find an orthonormal basis for the subspace spanned by $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$. There is more space for this on the next page too.

Continuation of problem (2).

(3) (3 points) Find the orthogonal projection of the vector $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ to the subspace of problem 2.

For the next six problems we will work in $P_1 \subset P_2 \subset C[-1,1]$. (Recall that $C[-1,1]$ is the set of real valued continuous functions defined on the interval $[-1,1]$.) Please note the -1 and the 1. They are important.

(4) (3 points) What is our usual inner product on $C[-1,1]$?

(5) (3 points) Find an orthonormal basis for P_1.

(6) (3 points) What is the orthogonal projection of x^2 to the subspace P_1.

(7) (3 points) Extend the orthonormal basis of problem (5) to P_2.

(8) (3 points) Find the 3×3 matrix for the orthogonal projection of P_2 to $P_1 \subset P_2$ with respect to the orthonormal basis you found in problem (5).

(9) (3 points) Find the 3×3 matrix for the orthogonal projection of P_2 to $P_1 \subset P_2$ with respect to the basis $\{x^2, x, 1\}$.

(10) (3 points) What is the volume of the parallelepiped determined by the vectors $(1, 0, 1)$, $(0, 2, 1)$, and $(1, 1, 0)$?

(11) (3 points) What is the definition of an orthogonal matrix?

(12) (2 points) What can you say about the columns of an orthogonal matrix?

(13) (2 points) What is the inverse of an orthogonal matrix?

(14) (3 points) Calculate the determinant of an orthogonal matrix.

(15) (3 points) If the determinant of a square matrix, $(v_1, v_2, \cdots, v_n)$, (i.e. with columns v_i), is 3, then what is the determinant of $(5v_1, v_2, \cdots, v_n)$?

(16) (3 points) Let A be an $n \times n$ upper triangular matrix and B be the same as A except that we have multiplied all of the diagnonal elements by 5. If the determinant of A is 3 then what is the determinant of B?

(17) (3 points) Find an orthonormal basis for the orthogonal complement of the subspace of $\mathbb{R}^3$ spanned by $(1, 1, 1)$. You can use the next (blank) page too.

Continutation of problem (17).