Exam #2, Linear Algebra, Spring, 2003, W. Stephen Wilson

Name: ________________________________

TA Name and section: ________________________________

NO CALCULATORS.

We work in $C[a,b]$, the continuous functions on $[a,b]$, for various a and b. We have an inner product, $< f,g >= \int_a^b f(x)g(x)dx$. Recall that P_n is the subspace of polynomials of degree $\leq n$.

I recommend being very careful and being sure you are right. You should check your work if possible. Work the problems you think are easiest first.

(1) (3 points) Show the set $V \subset P_3 \subset C[-1,1]$ defined as all $f \in P_3$ with $f(-1) = 0$ and $\int_{-1}^{1} f(x)dx = 0$ is a linear subspace of P_3.

(2) (5 points) Find a basis for the linear space in (1).

This series of problems will analyze the function x^2 by finding its linear approximation using least squares. DO NOT USE CALCULUS III to solve the problem, i.e., no partial derivatives.

We work in $P_1 \subset P_2 \subset C[0,1]$ with the usual inner product.

(3) (3 points) Find an orthogonal (doesn't have to be orthonormal) basis for P_1.

(4) (3 points) Find an orthonormal basis for P_1.

(5) (5 points) What is the orthogonal projection of $x^2 \in P_2$ onto $P_1 \subset P_2$, i.e. $\text{proj}_{P_1}(x^2)$? (Show work.)

(6) (3 points) Using the basis $\{x^2, x, 1\}$ for P_2, find the 3×3 matrix for $\text{proj}_{P_1} : P_2 \to P_1 \subset P_2$.

(7) (3 points) What is the dimension of the kernel of this linear transformation (in problem (6))? (Explain.)

(8) (3 points) Find a polynomial basis for the kernel in (7).

(9) (3 points) Find an orthogonal (not necessarily orthonormal) basis for P_2 that extends your orthogonal basis for P_1 from problem (3).

(10) (3 points) The polynomial of problem (5) minimizes a certain *least squares* integral. What is that integral?

(11) (3 points) If $\det(A) = 5$ and A is an $n \times n$ matrix, then what is $\det(5A)$? (Explain a bit.)

(12) (1 point) If A is an orthogonal rotation $n \times n$ matrix, then what is $\det(5A)$? (Explain a bit.)

(13) (3 points) State Cramer's rule.

(14) (3 points) What is the area of the parallelogram defined by the vectors $(1, 5)$ and $(3, 9)$.

(15) (3 points) If A is invertible, what is A^{-1} in terms of the adjoint (which you should define of course)?